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Opportunistic maintenance under periodic inspections in heterogeneous complex systems

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Abstract: A complex system consisting of monitored and non-monitored components is studied. Monitored components are subject to a degradation process, following a homogeneous gamma process. They are subject to a condition-based maintenance: the system is periodically inspected, and if the degradation level of a monitored component reaches a preventive threshold, the component is replaced by a new one. Furthermore, non-monitored components can fail between inspections. Time between these sudden failures follows an exponential distribution. Failures are self-announcing and the repair of the failed component is performed after a fixed delay time. In turn, these repair times are opportunities for preventive maintenance of the monitored components. Assuming a cost for each maintenance action, the expected cost rate of this system is analytically obtained. Numerical examples are given considering identical and non-identical components. Preventive thresholds and time between inspections that minimize the expected cost rate are evaluated.

Resumen: Se estudia un sistema complejo formado por componentes monitorizadas y no monitorizadas. Las componentes monitorizadas están sujetas a un proceso de degradación gamma homogéneo. Están sujetas a un mantenimiento basado en la condición del sistema: el sistema es inspeccionado periódicamente, y si el nivel de degradación de una componente alcanza el umbral preventivo, dicha componente es reemplazada por una nueva. Además, las componentes no monitorizadas pueden fallar entre inspecciones. El tiempo entre estos fallos sigue una distribución exponencial. Los fallos son *self-announcing*, y la reparación de las componentes estropeadas se realizan después de un tiempo de retraso fijado. De hecho, estos tiempos de reparación son oportunidades para realizar un mantenimiento preventivo de las componentes monitorizadas. Asumiendo un coste para cada acción de mantenimiento, se obtiene analíticamente el coste esperado de este sistema. Se muestran ejemplos numéricos considerando componentes idéndicas o no. Se evalúan los umbrales preventivos y el tiempo entre inspecciones que minimizan el coste esperado.

Keywords: maintenance, gamma process, monitored component, optimization.

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1. Introduction

A system is a set of components with the aim of carrying out a certain function. Nevertheless, systems are affected by external and internal degradation.

One example of this internal degradation is the pitting corrosion, which consists in the appearance of pits simultaneously on a system. On the other hand, we also can find systems subject to external deterioration, such as the changes in some material due to the temperature or humidity. These external factors are considered as shocks, which result in traumatic failures.

Maintenance plays an important role in areas such as engineering, where the failure of the system leads to high costs and production downtime. Rausand and Hoyland [2] classified the maintenance tasks into two groups:

(i) **Corrective maintenance:** it is performed when a system is not working. The purpose of this maintenance policy is to return the system to a good condition in which it can perform its function properly.

(ii) **Preventive maintenance:** it is a planned maintenance performed when the system is working, in order to avoid downs of the system and prevent total failures. This maintenance policy can be divided in other classes, for instance,

- *Age-based maintenance:* maintenance actions are performed when the system exceeds a certain fixed age.
- *Condition-based maintenance:* this is also called *preventive maintenance*. With this policy, the system is maintained when its deterioration level exceeds a certain threshold.

In our model, condition-based maintenance is implemented through two different thresholds that control the state of the system: a preventive threshold (denoted by **M**) and a corrective threshold (denoted by **L**), lower than the previous one.

2. System description

The general assumptions of the model are the following:

(i) Monitored components of the system are subject to a continuous degradation, which follows a gamma process with shape and scale parameters α_i and β_i. Let $X_i(t)$ be the degradation of the monitored component i at time t. Its density function is given by:

$$f_{\alpha_i(t),\beta_i} = \frac{\beta_i^{\alpha_i t}}{\Gamma(\alpha_i t)} x^{\alpha_i t-1} \exp\{-\beta_i x\}, \quad x \geq 0,$$

where $\Gamma(\cdot)$ is the well-known gamma function.

(ii) Non-monitored components represents the sudden shocks to which the system is subject. Failures arrivals are exponentially distributed, that is, they follow a Poisson arrival process. Let Y be the time between these failures, then the survival function of Y is given by:

$$\bar{F}_Y = \exp\{-\lambda t\},$$

where λ is the parameter of the underlying Poisson process. Notice that non-monitored components can only be maintained upon failure, and we cannot predict when the failure will occur.

(iii) Failures of both monitored and non-monitored components are independent. When a component fails, a signal is immediately sent to the repair time, and it arrives with a delay of τ time units to start the reparation.

(iv) The system is subject to periodic inspections, that is, the deterioration level of the monitored components is checked each T time units, which is the inspection period.

(v) An opportunistic maintenance policy is implemented on the system: maintenance and inspection times of the system are seen as opportunities to check the state of the rest of the monitored components and perform a preventive maintenance of them if necessary.

3. Mathematical modelling

A *system renewal* is the maintenance time in which all the monitored components are replaced and the time to the next inspection is T time units. However, describing the state using renewal theory is complicated, since between renewals many preventive replacements can occur. To deal with it, semi-regenerative processes are used instead of renewal processes. A *semi-regenerative cycle* is defined as the time between two successive maintenance actions (which are the semi-regeneration points).

With that, we are able to study the evolution of the system with a Markov chain.

Let O_k be the time between the $(k-1)$-th and the k-th maintenance actions. The multiple process

$$(X_1(O_k), X_2(O_k), \dots, X_m(O_k))$$

is a Markov chain with state space $[0, M) \times \overset{m)}{\cdots} \times [0, M)$.

If the previous chain comes back to the initial state $(\mathbf{0}, \dots, \mathbf{0})$ almost surely (that is, is a regeneration point), then there exists a stationary measure π solution of the equation

$$\pi(\cdot) = \int_0^M \int_0^M \dots \int_0^M \mathbb{Q}(\cdot|\mathbf{x})\pi(\mathrm{d}\mathbf{x}), \tag{1}$$

where $\mathbb{Q}(\cdot|\mathbf{x})$ denotes the *kernel* of the process.

A result that assures a finite expected time to the system renewal is given. The proof can be seen in Proposition 4.1 of [1].

Lemma 1. *If $\mu < 1$, where μ is*

$$\mu = 1 - F_Y(T-\tau) \prod_{i=1}^{m} (F_{\alpha_i \tau, \beta_i}(M)) F_{\alpha_i (T-\tau), \beta_i}(L-M),$$

then the stationary distribution π in (1) exists.

With the existence of the stationary measure π, the state of the system can be described at any time, so we can study now the objective function of the model.

4. Optimization of the objective function

Theorem 2. *For any realisation of the process, the long-run average reward per time unit is equal to the expected reward earned during one cycle divided by the expected length of one cycle. That is,*

$$P\left[\lim_{t \longrightarrow \infty} \frac{\mathbb{E}[C(t)]}{t} = \frac{\mathbb{E}[C(O_1)]}{\mathbb{E}[O_1]}\right] = 1,$$

where O_1 stands for the time to the next maintenance (that is, the length of a maintenance cycle) and $C(t)$ is the total cost at time t.

Each maintenance task implies a certain cost. Let C_∞ be the asymptotic cost rate. With the renewal-reward theorem, the cost can be developed as

$$C_\infty(T, M) = \frac{\mathbb{E}[C^c(O_1)]}{\mathbb{E}[O_1]} + \frac{\mathbb{E}[C^p(O_1)]}{\mathbb{E}[O_1]} + \frac{\mathbb{E}[C^{nm}(O_1)]}{\mathbb{E}[O_1]} + \frac{\mathbb{E}[C(I(O_1))]}{\mathbb{E}[O_1]} + \frac{\mathbb{E}[C(D(O_1))]}{\mathbb{E}[O_1]} - \frac{\mathbb{E}[R(O_1)]}{\mathbb{E}[O_1]},$$

where $\mathbb{E}[C^c(O_1)]$ and $\mathbb{E}[C^p(O_1)]$ are the expected costs due to preventive and corrective maintenance of monitored components in a cycle, respectively; $\mathbb{E}[C^{nm}(O_1)]$ denotes the expected cost due to the corrective replacement of non-monitored components; $\mathbb{E}[C(I(O_1))]$ corresponds to the expected cost due to inspections; $\mathbb{E}[C(D(O_1))]$ is the expected cost due to downtime, and $\mathbb{E}[R(O_1)]$ stands for the expected reward obtained in a semi-regenerative cycle.

$$C(T_{opt}, M_{opt}) = \inf\{C_\infty(T, M), T < 2\tau, M < L\}.$$

The following sequence of costs is used to study our maintenance strategy:

- Corrective replacement cost of monitored component i: $C_i^c = 80$ monetary units, for all $i \in I$.
- Preventive replacement cost of monitored component i: $C_i^p = 30$ monetary units, for all $i \in I$.
- Corrective replacement cost of non-monitored components: $C^f = 80$ monetary units.
- Downtime cost of monitored component i: $c_i = 5$ monetary units per time unit, for all $i \in I$.
- Downtime cost of non-monitored components: $c^{nm} = 5$ monetary units per time unit.

Furthermore, a reward provided by the monitored components of the system is considered. It depends on the deterioration level of the monitored components, and it decreases as the deterioration level of a component increases, so a classical exponential function is used to model it. Given the deterioration level x of the monitored component i, the reward function r_i is

$$r_i(x) = \theta_0 + g\exp\{-\gamma_i x\}, \quad 0 \le x \le L, \quad \gamma_i > 0, \ \ \forall\, i,$$

where θ_0 and g are constants greater than 0.

To deal with the optimization of the objective cost function, we propose the following method. Firstly, typical Monte-Carlo simulation is used to search for potential solutions of the optimal values of the time between inspections (or inspection period) T and the preventive threshold, denoted by M.

After that, some meta-heuristic algorithms, such as *Pattern Search* and the *Genetic Algorithm*, are employed to optimize the previous parameters T and M. Nowadays, meta-heuristics are widely employed in stochastic problems, to provide a sufficiently good solution to an optimization problem, despite the fact that they don't guarantee a globally optimal solution in some problems.

The results obtained with this method are shown in Table 1:

m	T_0	M_0
2	4.39	2.95
3	4.28	3.17
4	3.13	3.38
5	2.47	3.78
6	2.29	3.93
7	1.89	4.18
8	1.64	4.37
9	1.53	4.65
10	1.48	4.70

Table 1: (a) Starting points with MC.

m	(T_{opt}, M_{opt})	$C_\infty(T_{opt}, M_{opt})$
2	(5.20, 2.02)	8.94
3	(3.86, 2.40)	11.02
4	(3.19, 2.88)	13.05
5	(3.20, 2.38)	13.10
6	(2.71, 3.51)	16.44
7	(2.30, 3.01)	16.90
8	(1.73, 3.22)	18.84
9	(1.46, 3.36)	20.11
10	(1.19, 3.45)	21.45

(b) T_{opt}, M_{opt} and C_∞ using the *Pattern Search*.

References

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