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Evolution of mathematical modelling as a method of scientific cognition and its didactic functions in educational process

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Abstract: We investigate historical aspects of developing mathematical modelling method and analyze the functions of mathematical modelling in the process of cognition. It is justified that modelling in the educational process acts simultaneously as a method of scientific knowledge, a part of the content of educational material and an effective mean of its study.

We show that the development of students' ideas about the role of mathematical modelling in scientific knowledge and practice, the development of their ability to build mathematical models of life phenomena, are an important task of modern school. A possibility of improvement pupils' literacy in mathematics is highlighted by developing their correct conceptions about the method of mathematical modelling.

Resumen: Investigamos los aspectos históricos del desarrollo del método de modelización matemática y analizamos las funciones de la modelización matemática en el proceso de cognición. Se justifica que la modelización en el proceso educativo actúa simultáneamente como un método de conocimiento científico, una parte del contenido del material educativo y un medio eficaz de su estudio.

Mostramos que el desarrollo de las ideas de los alumnos sobre el papel de la modelización matemática en el conocimiento y la práctica científica, el desarrollo de su capacidad para construir modelos matemáticos de los fenómenos de la vida, son tareas importantes de la escuela moderna. La posibilidad de mejorar la alfabetización matemática de los alumnos se pone de manifiesto en el desarrollo de sus concepciones correctas sobre el método de modelización matemática.

Keywords: model, mathematical modelling, cognition.

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1. Introduction

Mathematical modelling is one of the main modern methods of reality cognition. It is widely used in all areas of research.

The purpose of our research is to consider historical aspects of the formation of the modelling method and to highlight its epistemological functions.

There are various interpretations of a model and modelling process concepts in modern scientific literature. A term "model" is understood as a mental or materially realized system, which is able to replace an object of study by reflecting or reproducing it, so that the study of model provides new information about this object (see V. A. Shtoff [5]). Accordingly, modelling is a scientific method of studying different systems by building models of these systems, which preserve some of the main features of the subject of study, and a study of functioning of models with the transfer of data to the subject of research.

The mathematical model of the system is understood as a set of relations (formulas, equations, inequalities, etc.), which determine characteristics of states of the system depending on its parameters, external conditions, initial conditions and time. By the definition of V. M. Glushkov, a mathematical model is a set of symbolic mathematical objects and the relations between them. For M. M. Amosov [1], a mathematical model is a system that reflects another system.

The term "model" covers an extremely wide range of material and ideal objects. Determining the epistemological role of modelling theory, that is, its significance in the process of cognition, it is necessary, first of all, to start from the historical aspects of the formation of the modelling method.

2. Historical aspects of mathematical modelling

An investigation of historical aspect of developing mathematical modelling method shows that the progress is closely linked to the development of a science.

Mathematical modelling originated in ancient times. Ancient Greek philosophers, for example, Archimedes (287 – 212 B.C.), Democritus (460 – 370 B.C.) or Epicurus (341–270 B.C.), have already explained physical properties of objects creating some analogies on intuitive base. The appearance of experimental modelling was connected with the names of Leonardo da Vinci (1452 – 1519), Johannes Kepler (1571 – 1630), Galileo Galilei (1564 – 1642), Nicolaus Copernicus (1473 – 1543). The scientists applied analog models, created graphic constructions from real objects and obtained results in further research. It was Galileo Galilei who proved the inability to bring a similarity of mechanical systems to their geometric similarity. He also claimed that conclusions obtained from only geometrically similar to its prototype models often lead to mistakes.

A powerful force for the mathematical modelling development was the appearance of mathematical notation systems. The historically first comprehensive mathematical model was the classical mechanics of Isaac Newton (1642 –1727). The scientist has initiated modelling method as those of theoretical research.

The term "model" was introduced to mathematics in the 19th century because of an emergence of the hyperbolic geometry of Nikolai Lobachevsky (1792 – 1856) and the spherical geometry of Georg Riemann (1835 – 1900). The term was later used by the German mathematician Felix Klein (1849 – 1925). Nevertheless, an application of mathematical modelling in the 19-20th centuries was accompanied by certain difficulties. It happened so because of the lack of researcher's mathematical education to describe mathematically new phenomena of science. Therefore, mathematical modelling was applied only in those branches of knowledge that have gained a high level of development. The first mathematized science is physics. The example of physics mathematization shows a parallel development of both sciences. However, in some cases mathematics has left physics behind preparing a necessary apparatus for it.

In the latter half of the 20th century there appeared a great value of works investigating mathematical modelling in epistemological and didactic aspects. A fundamental research in this field was conducted by the following scientists: N. H. Alekseev, B. M. Kedrov, V. A. Shtoff, A. I. Uyemov, L. M. Fridman, L. R. Kalapusha, etc.

There are various classifications of models in modern scientific literature. Scientists highlight functions of modelling, analyze the connections of modelling with other experimental and theoretical methods of cognition (see V. E. Bakhrushin [1], L. R. Kalapusha [3], B. A. Glinskii, B. S. Gryaznov, B. S. Dynin and E. P. Nikitin [2]).

It is shown that a mathematical model can be created in three ways:

(i) as a result of direct study of the real process (phenomenological model);

(ii) as a result of the deduction process, when the new model is a special case of a certain general model (asymptotic model);

(iii) as a result of the induction process, when the new model is a generalization of other models (ensemble models).

The model, as a special epistemological form, can be understood only when considering the set of its various functions. The analysis of scientific and methodical literature showed the diversity of views of scientists on the definition of epistemological functions of the modelling method.

V. A. Shtoff [5] states: "In the theoretical thinking one see a domination of the one side, in sensory perceptions and observations — the other, whereas in the model they are linked together, and in this regard we have a specifics of the model and one of its most important epistemological functions."

I. Novik [4] distinguishes five main functions of modelling:

(i) illustrative,

(ii) translational,

(iii) substitution-heuristic,

(iv) approximation,

(v) extrapolation-prognostic.

The scientist notes that these functions are not alternative, they coexist in models, but their presence in each model is optional; moreover, some other functions may be found in certain models.

A number of scientists (B. A. Glinskii, E. P. Nikitin and others, see [2]) distinguish such modelling functions as:

(i) interpretive (explanations based on logic and formalized language of presentation),

(ii) explanatory (shows that this object is a subject to a particular law or set of laws),

(iii) predictive (operation, the task of which is to obtain data on objects and processes or non-existent, or existing, but not known), and

(iv) criterion (verifying the truth of knowledge about the original).

We can undoubtedly state that mathematical modelling as a method of reality cognition is used not only because it can replace an experiment. It has a great independent significance because

(i) with the help of mathematical modelling, it is possible to develop different mathematical models on the basis of the same data, and these models would interpret the studied phenomenon differently;

(ii) in the process of model building, one can make various additions to the hypothesis under study and get simplification;

(iii) in the case of complex mathematical models, one can use computers;

(iv) it is possible to conduct model experiments.

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It was established that an acquaintance of pupils (students) with the method of modelling, including mathematical, helps them to understand logic of scientific knowledge and to learn its methodology (see the book by L. R. Kalapusha [3]).

Modelling in the educational process acts simultaneously as:

(i) a method of scientific knowledge,

(ii) a part of the content of educational material, and

(iii) an effective mean of its study.

The development of students' ideas about the role of mathematical modelling in scientific knowledge and practice, the development of their ability to build mathematical models of life phenomena, are an important task of modern school.

In particular, we need to pay special attention to developing students' skills to reformulate an applied problem into the language of mathematics and to create adequate mathematical models. It is important, that students concentrate correctly, highlighting the essential and non-essential properties of objects; abstract from insignificant properties; correctly interpret the relationships between objects of the problem. The teachers should form a particular attitude of students to the acquired knowledge through the disclosure of the essence of mathematical modelling.

Despite the widespread use of the method of mathematical modelling, the development of relevant students' skills is not systematic and is mostly done during mathematics lessons. This significantly reduces didactic effectiveness of the use of this method in learning process, in particular, in increasing mathematical literacy of students. To overcome this limitation, in our opinion, it is possible to use interdisciplinary connections more effectively.

For example, when generalizing the basic properties of directly proportional and linear functions in mathematics lessons, it is advisable to use the knowledge of students that were already obtained in physics lessons when studying thermal phenomena. We offer them, as a homework, to build graphs of the amount of heat obtained during the combustion of this type of fuel, its mass, for example, for dry firewood, anthracite, gasoline. We analyze these graphs in math lessons. It is possible not only to repeat the basic properties of direct proportionality, but also to form in students the concept of function as a mathematical model.

The development of correct ideas of students about the nature of the reflection of mathematical phenomena and processes of the real world, the role of mathematical modelling in scientific knowledge and in practice, is of great importance for the formation of their mathematical literacy.

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